The Mathematics of Radioactive Decay

1) Discovery of the radioactive decay law. In 1900, Ernest Rutherford noticed a decrease over time in the radioactive intensity of a sample he was studying, so he begin to measure this decrease. He determined that the decrease fit the following formula (using modern symbols):

$$N = N_o e^{-kt} \qquad (1)$$

where N = the amount of sample remaining after an amount of time 't' and N_o = the original starting amount of the sample. e has its usual meaning and k is a constant with a unique value for each substance. Rutherford named it the radioactive decay constant.

2) Straight-line form of the decay law. Equation 1 can be modified to the form of a straight line formula (y = mx + b) as follows:

$N / N_o = e^{-kt}$ by rearrangement

$\ln (N / N_o) = -kt$ take natural logarithm of each side (2)

$\ln N - \ln N_o = -kt$ division of exponents is done by subtraction

$\ln N = -kt + \ln N_o$ by rearrangement

This last equation fits the general form of a straight-line equation. The y-axis is the natural log of N and the x-axis is t (time). The line will from upper left to lower right (negative slope) with negative k being the slope.

3) An equation for calculating half-life. The equation (2) can be modified to give an equation where knowing the value for k leads directly to the length of the half-life.

$$t = \frac{-\ln (N / N_o)}{k}$$ by rearrangement

From the definition of half-life, we obtain the value for N / N_o as one-half. After one half-life, half the material on-hand at the start will have decayed. If the starting amount equals 2, then the ending amount one half-life later will equal one.

Considering the numerator only: $-\ln (1/2) = -(\ln 1 - \ln 2) = -(0 - \ln 2) = \ln 2$

Letting $T_{1/2}$ = time of one half-life, we obtain:

$T_{1/2} = 0.693 / k$ since ln 2 = 0.693

4) Miscellaneous half-life information. The fraction one-half figures prominently in this.

$(1/2)^n$ where n = the number of half-lives yields the decimal portion of substance remaining.

If the decimal fraction of substance remaining is known, setting it equal to $(1/2)^n$ and solving will yield the number of half-lives elapsed.